

A Cubic Micron of Equilibrium Pair Plasma?

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ABSTRACT

Is it possible to create a small volume of equilibrium pair plasma in the laboratory with an intense laser pulse? No. The required power would exceed 10^{18} W. The number of seed particles required to absorb the laser energy would approach the number of pairs in the desired plasma, and their radiation in a visible laser field would occur at excessively high frequencies.

Subject headings: Plasmas

1. Introduction

Space in thermal equilibrium at sufficiently high temperature is filled with an electron-positron pair plasma. If there are no baryons present the equilibrium densities of electrons and positrons are equal and are given by

$$n_{\pm} = \frac{1}{\pi^2 \hbar^3} \int_0^{\infty} \frac{p^2 dp}{\exp(p^2 + m^2 c^4)^{1/2} / k_B T + 1}, \quad (1)$$

where m is the electron's mass and other symbols have their usual meanings (Landau & Lifschitz 1958). If $k_B T \ll mc^2$ this result is approximated

$$n_{\pm} \approx 2 \left(\frac{mk_B T}{2\pi \hbar^2} \right)^{3/2} \exp(-mc^2/k_B T). \quad (2)$$

A pair plasma with this density is in complete thermodynamic equilibrium with the photon gas into which it annihilates and from which it is formed by pair production, and each species has zero chemical potential. The radiation field has a Planckian spectrum at temperature T .

It is necessary to distinguish this pair plasma in equilibrium with a black body radiation field from the much lower density pair plasmas frequently considered in astrophysics. These low density pair plasmas may have equilibrium relativistic Maxwellian particle distribution functions, but their densities are lower, usually by many orders of magnitude, than would be given by Eq. 1. Their chemical potentials are negative and many times mc^2 in magnitude. They are not in equilibrium with a Planckian radiation field. Their optical depth is generally very low, and the radiation energy density is much less than that of the pairs. Such low density pair plasmas have been considered by many authors in many astrophysical contexts, but will not be discussed further here.

Completely equilibrium pair plasmas occur (with the addition of baryons and a consequent shift in chemical potential and inequality between n_- and n_+) in the interior of very evolved stars, where they are hidden from view. Equilibrium pair plasmas have also been considered (Thompson & Duncan 1995, Katz 1996) as sources of emission from soft gamma repeaters (SGR), if trapped within the strong magnetic field of a neutron star. The minimum required magnetic field is only $\sim 10^{11}$ gauss, although much stronger fields

have been considered. Directly observed pair plasma would be expected to produce an approximately black body radiation spectrum, whose temperature is only weakly dependent on the many unknown astrophysical parameters. This is in at least qualitative agreement with observations of SGR, whose spectra appear to be strongly self-absorbed at low frequencies, to be characterized by temperatures roughly comparable to those predicted by pair plasma models, and which are nearly uniform from burst to burst and within bursts. In addition, an equilibrium pair plasma is such a simple state of matter, which occurs whenever the energy density is sufficient and the optical depth large, that it may be found in unanticipated places in the universe and is likely to repay study.

2. Equilibrium Pair Plasmas in the Laboratory?

2.1. Energetics

The advent of high power lasers, now including the Petawatt (10^{15} W) laser, raises the question of whether it might be possible to create a small volume of equilibrium pair plasma in the laboratory. The first criterion which must be satisfied is energetic: Can a laser supply the power lost from the surface of the plasma?

There are two mechanisms of energy loss. The surface will radiate approximately as a black body at its temperature, and the pairs themselves will expand freely (in contrast to the situation near a neutron star, the magnetic field required to confine the pairs is unachievable in the laboratory). It is easy to see that the second process is less important than the first, partly because the pairs escape at a slower speed than photons, and partly because the equilibrium pair energy density is always less than 7/4 that of photons at the same temperature, and is less by an exponentially large factor if $k_B T \ll mc^2$, the case of interest, because of the exponential in Eq. 2. We will therefore only consider energy loss by radiation.

The minimum temperature required to obtain an equilibrium pair plasma may be estimated by requiring that the Thomson scattering optical depth be at least 3, in order that photon-pair equilibrium be achieved. Assume a spherical plasma of radius $r = 0.6 \mu$, which has a volume of nearly a cubic micron. This size is chosen because it is approximately the size of a diffraction-limited focal spot of a visible laser. Then the required pair density, allowing for the presence of both species, is

$$n_{\pm} = \frac{3}{2\sigma_{es}r} \approx 4 \times 10^{28} \text{ cm}^{-3}. \quad (3)$$

This is probably an underestimate of the actual required density because of the decrease of cross-section with increasing energy and the incomplete equilibration achieved in Compton scattering in contrast to absorption.

The required temperature may be found from Eqs. 3 and 4, with the result $T = 2.3 \times 10^9$ °K and the parameter

$$u \equiv \frac{k_B T}{mc^2} = 0.39. \quad (4)$$

The resulting black body intensity radiated

$$I = \frac{2\pi^5}{15} \frac{m^4 c^6}{h^3} u^4 = 1.6 \times 10^{26} \text{ W/cm}^2. \quad (5)$$

The total power radiated

$$P = 4\pi r^2 I = 7 \times 10^{18} \text{ W}. \quad (6)$$

This power need be radiated only for a time $O(r/c) \sim 2 \times 10^{-15}$ s, so the required energy is only ~ 15 kJ. However, such brief illumination would require a laser of a bandwidth far broader than any known today.

These numbers are intimidating. The power exceeds that of the Petawatt laser by nearly four orders of magnitude. While this would not violate any fundamental physical bound, it is certainly beyond anything achievable within foreseeable budgets and with foreseeable technology.

Larger volumes of pair plasma would require even more power. First suppose $n_{\pm} \propto T^3$, as is the case for $k_B T \gg mc^2$. Then the temperature required (by the condition that the optical depth equal 3) $T \propto n_{\pm}^{1/3} \propto r^{-1/3}$. The radiated power $P \propto T^4 r^2 \propto r^{2/3}$ increases as r increases. In fact, in the applicable regime $k_B T \ll mc^2$ the decrease of T with decreasing n_{\pm} and increasing r is slower (the sensitivity of n_{\pm} to T is greater, because of the exponential in Eq. 2), and the dependence of P on r approaches the second power. It is clear that increasing r only increases the required power.

Values of r significantly less than 0.6μ are excluded, so long as power is provided by a laser of near-visible wavelength, by the diffraction limit on focal spot size. However, even if much smaller r could be obtained the required power would not be much less. The reason for this is that values of r less than a few tenths of a micron imply $k_B T > mc^2$, and the Thomson cross-section must be replaced by the Klein-Nishina cross-section, which decreases approximately $\propto T^{-1}$. Taking $n_{\pm} \propto T^3$ yields $r \propto T^{-2}$ and $P \propto T^4 r^2$, independent of T and r .

2.2. Kinetics

Suppose that, somehow, visible lasers of the required power were available. The implied electric and magnetic fields would be impressive, with amplitudes

$$B_0 = E_0 = \left(\frac{8\pi I}{c} \right)^{1/2} = \left(\frac{16\pi^6}{15} \right)^{1/2} \frac{m^2 c^{5/2}}{h^{3/2}} u^2 = 1.2 \times 10^{12} \left(\frac{u}{0.39} \right)^2 \quad (\text{cgs units}). \quad (7)$$

These should be compared to the characteristic quantum fields at which Landau levels are spaced by mc^2 and spontaneous electric pair production is rapid $B_q = E_q \equiv m^2 c^3 / e \hbar$:

$$v \equiv \frac{E_0}{E_q} = \left(\frac{2\pi^3}{15} \alpha \right)^{1/2} u^2 = 0.026 \left(\frac{u}{0.39} \right)^2, \quad (8)$$

where $\alpha \equiv e^2 / \hbar c$ is the usual fine-structure constant.

We now ignore the possibility of spontaneous pair production, and consider only the effects of such large fields on an injected electron or a small number of pairs. Will their motion lead to rapid radiation of the soft gamma-rays which can create the desired equilibrium pair plasma? We consider only the radiation by individual particles in intense fields, and also ignore such processes as inverse bremsstrahlung, which are generally slow at high energies and low densities.

The theory of the motion of a charged particle in a strong electromagnetic wave was recently reviewed by Meyerhofer 1997, and many useful results are presented by Gunn & Ostriker 1971. I will apply results which assume a plane space-filling electromagnetic wave. In fact, these results overestimate by a very large factor the energy absorbed in a small laser focal spot by charged particles which soon leave the region of high intensity.

The intensity of an electromagnetic field of angular frequency ω and wavelength $\lambda = 2\pi c/\omega$ is described by a parameter

$$\eta \equiv \frac{eE_0}{mc\omega} = 5500 \left(\frac{u}{0.39} \right)^2 \frac{\lambda}{0.5 \mu}. \quad (9)$$

Radiation reaction is described by a parameter

$$\epsilon \equiv \frac{e^2 \omega \eta^2}{3mc^3} = 0.35 \left(\frac{u}{0.39} \right)^4 \frac{\lambda}{0.5 \mu}. \quad (10)$$

The radiation parameter is much larger than that found for pulsar fields, for which $\epsilon \ll 1$, but is small enough that the results of Gunn & Ostriker 1971 are still approximately valid. The mean power radiated per electron (or positron) in the laser field is given by Gunn & Ostriker 1971, in a process which they name nonlinear inverse Compton scattering,

$$P_r = \frac{e^4 E_0^2}{3m^2 c^3} = 110 \left(\frac{u}{0.39} \right)^4 \text{ W}. \quad (11)$$

Approximately 6×10^{16} electrons and positrons must be present to absorb the power required (Eq. 6). This is approximately equal to the 8×10^{16} particles present in a cubic micron of equilibrium pair plasma (Eq. 3). In other words, the laser could sustain the plasma once it was created, but could not produce it from a much smaller number of seed particles by nonlinear inverse Compton scattering.

Gunn & Ostriker 1971 also estimate the critical frequency ν_{crit} of nonlinear inverse Compton scattering for a very energetic particle (Lorentz factor $\gg \eta mc^2$) entering the laser field. Their argument may also be applied to particles accelerated by the laser field itself, with similar results:

$$\nu_{crit} \approx \omega \eta^3 \approx \frac{v^3}{w^2} \frac{mc^2}{\hbar} \approx 10^{26} \left(\frac{u}{0.39} \right)^6 \left(\frac{\lambda}{0.5 \mu} \right)^2 \text{ Hz}, \quad (12)$$

where the parameter $w \equiv \hbar \omega / mc^2$.

These are photons of $\sim 10^{11}$ eV energy. Their cross-section for photon-photon pair production is very small, and they are produced (because of their high energy) in very small numbers even if sufficient charged particles were present to absorb the required power. If produced they will escape, rather than producing an equilibrium pair plasma.

3. Discussion

It is apparent that near-visible lasers cannot be used to produce an equilibrium pair plasma, even if sufficient power were available. This goal requires not only extremely high power (Eq. 6) but also that this power be delivered in photons of energy approaching mc^2 (Eq. 12). Until gamma-ray lasers are developed, this is likely to remain unachievable.

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